Learning, enjoying and growing together!

The Woodlands Community Primary School

Woodlands Calculation Progression

<u>Written Calculation Progression</u>

This document maps written formal calculation methods for addition, subtraction, multiplication and division. Written Calculation Progression links the key concrete experiences with pictorial and abstract representations. This supports pupils to move with confidence and deep conceptual understanding through each strand of calculation.

The Importance of Mental Mathematics

While this policy focuses on written calculation in mathematics, we recognise the importance of mental strategies and known facts that form the basis of all calculations. A range of mental strategies are developed in a separate mental maths progression map. Both mental and written calculations will intertwine with one another, as key number facts become embedded, leading to greater ease of using a written method.

Concrete, Pictorial and Abstract

Concrete manipulatives

Concrete manipulatives are objects that can be touched and moved by pupils to introduce, explore or reinforce a mathematical concept. They provide a vehicle to help pupils make sense of complex, symbolic and abstract ideas through exploration and manipulation. Furthermore, they support the development of internal models and help build stronger memory pathways.

the same end results.

Practising concrete and pictorial methods will lead on to the final written methods (the abstract) and should be used prior to embedding the written method. This will give children a firmer understanding of these procedures and the ability to reason their working out.

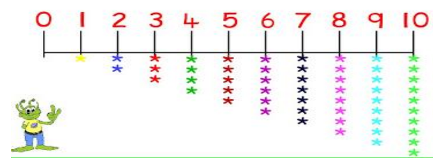
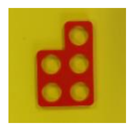
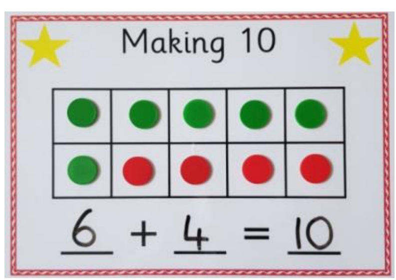
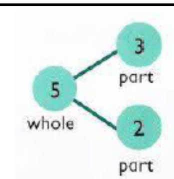
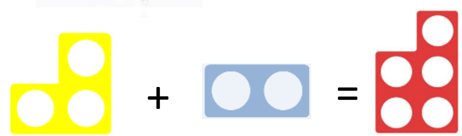
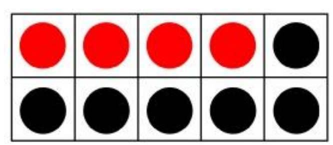
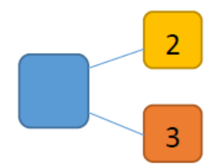
Pictorial (including jottings)
The act of translating the concrete experience into a pictorial representation helps focus attention on what has happened and why. This supports deeper understanding and a stronger imprint on memory. Pictorial representations are more malleable than concrete resources and, once understanding is secured, allow exploration of complex problems that may be challenging to reproduce with manipulatives.

Abstract - Written
The aim, within this policy, is for compacted forms of notation. These have developed through the history of mathematics. Explicit individual steps in procedure are hidden or they have been shortcut. The informal and expanded methods expose all the intermediate steps, replicating thought processes more closely and support understanding prior to compaction.

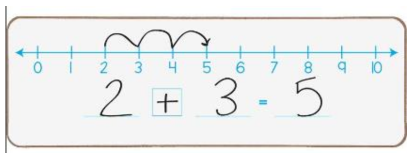
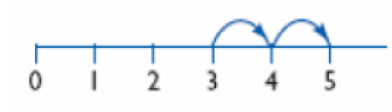
Abstract – Spoken
Learning to use the correct mathematical vocabulary is vital for the development of mathematical proficiency. The ability to articulate accurately allows pupils to communicate and build meaning. Ideas become more permanent.

As the document progresses, the examples given in the concrete and pictorial sections use different manipulatives e.g. dienes, counters, ten frames, Numicon. This is to demonstrate that the different written methods used, can be practised with a range of different resources leading to the same end results.

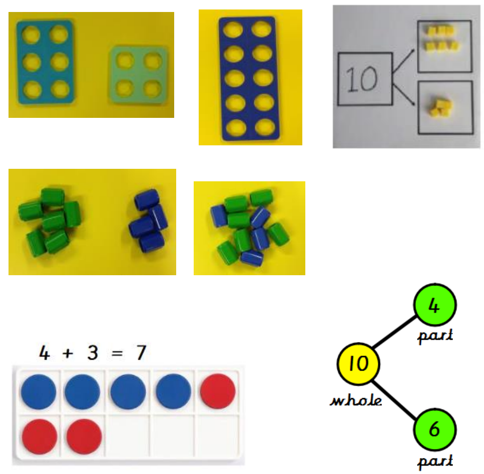
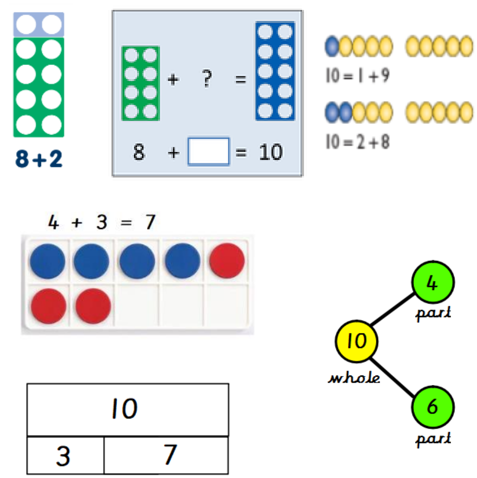
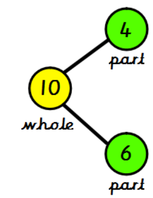
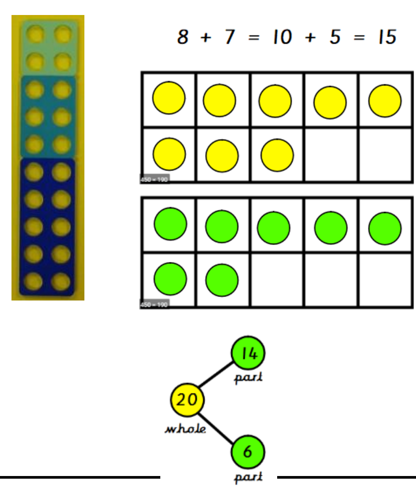
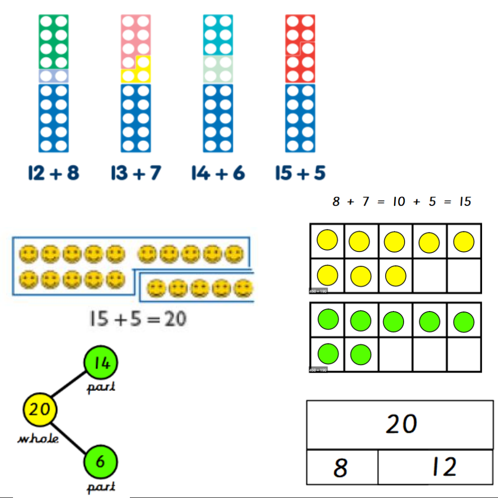
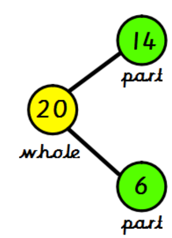
Woodlands Calculation Policy
Addition

	Objectives/stages in development	Concrete	Pictorial	Abstract
Reception	Numbers as labels for counting		  	0 1 2 3 4 5 6 7 8 9 10
	Add two single digit numbers	     	   	2 + 3 = 5 3 + 2 = 5 5 = 3 + 2 5 = 2 + 3 

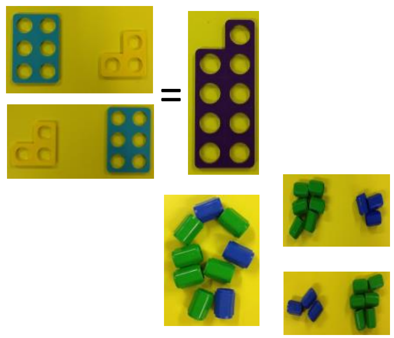
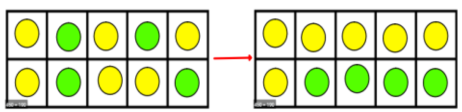
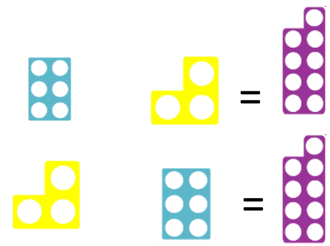
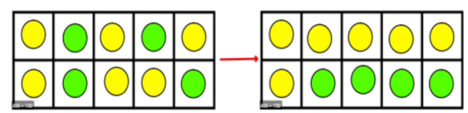
Woodlands Calculation Policy
Addition

	Objectives/stages in development	Concrete	Pictorial	Abstract
Reception	Counting	 		3 + 2 = 5

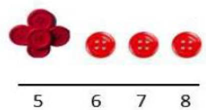
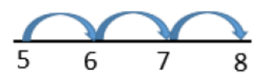
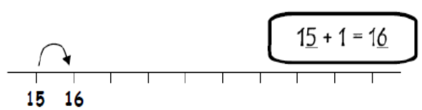
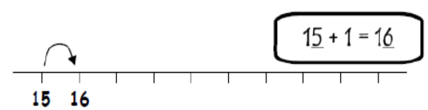
Woodlands Calculation Policy
Addition

<table>
<tr><th></th><th>Objectives/ stages in developme nt</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr><td rowspan="2">Year 1/Year 2 (Year two to achieve rapid recall of number facts and bonds.</td><td>Number bonds of 10
Number Facts up to 10</td><td>
</td><td>
</td><td>4 +6 = 10
6 + 4 = 10
10 = 6 + 4
10 = 4 + 6

</td></tr>
<tr><td>Number bonds of 20
Number facts up to 20</td><td>
</td><td>
</td><td>14 +6 = 20
6 + 14 = 20
20 = 6 + 14
20 = 14 + 6

</td></tr>
</table>

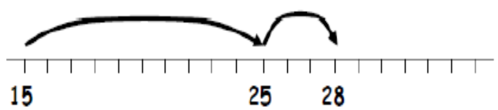
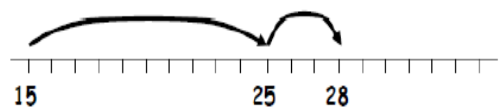
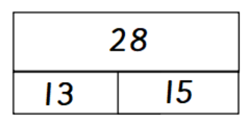
Woodlands Calculation Policy
Addition

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 1	Regrouping (know addition can be done in any order)	  6 + 4 =10	  6 + 4 =10	6 + 3 = 9 3 + 6 = 9

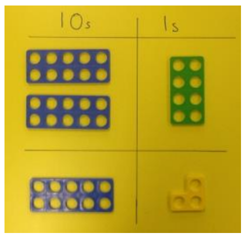
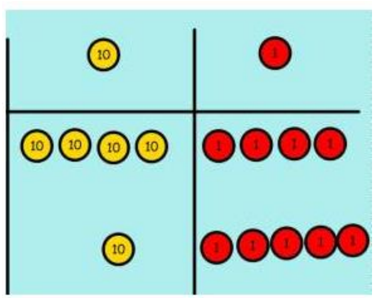
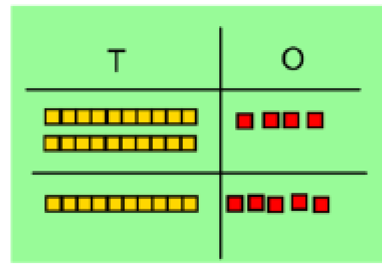
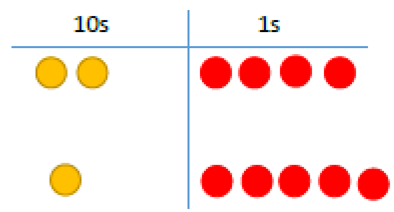
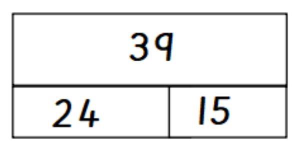
Woodlands Calculation Policy
Addition

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 1	Counting	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. 	Use a number line to count on in ones. 	5 + 3 = 8
Year 2	Counting on	Counting on in ones Counting on in tens Counting on in both tens and ones 		$\boxed{15 + 13 = 28}$

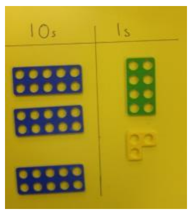
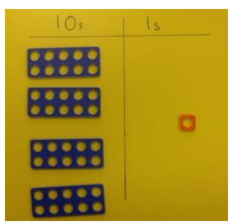
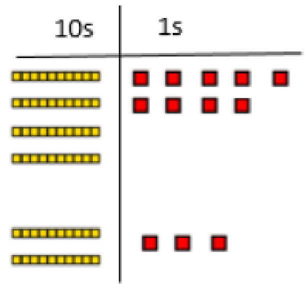
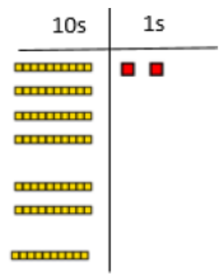
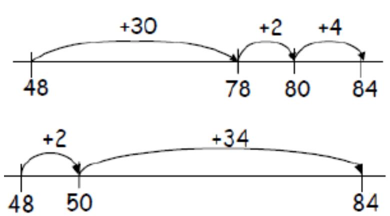
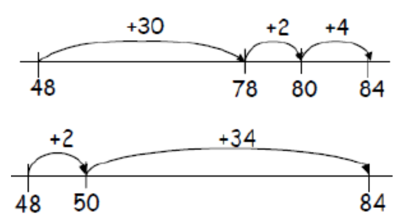
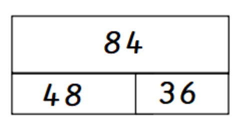
Woodlands Calculation Policy
Addition

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 2	Counting on	 	  	15 + 10 = 25 15 + 13 = 28

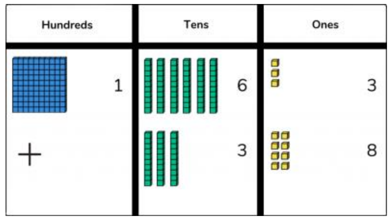
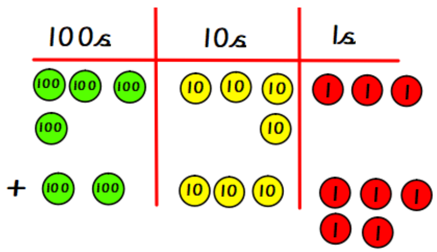
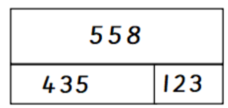
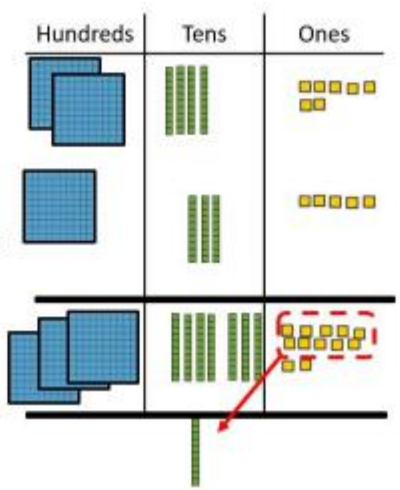
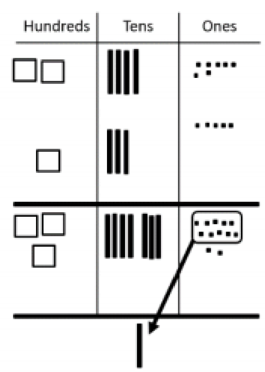
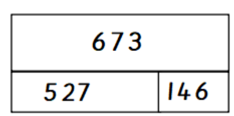
Woodlands Calculation Policy
Addition

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 2	Column method without regrouping	 	  	24 + 15 = 39 24 + 15 39

Woodlands Calculation Policy
Addition

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 2	Column method with regrouping	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for 1 ten. 	 	40 + 9 20 + 3 60 + 12 = 72
Year 3	Counting on		 	48 + 36 = 84

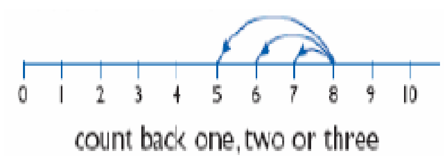
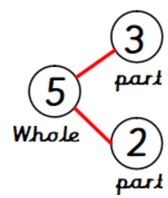
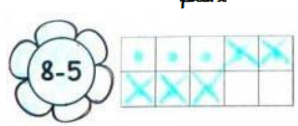
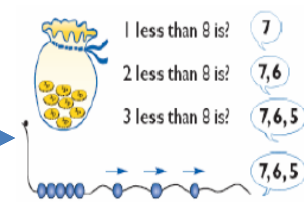
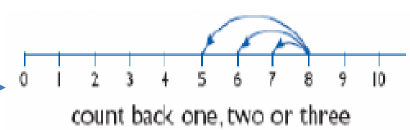
Woodlands Calculation Policy - Addition

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 3	Column method without regrouping (3 digit numbers)		435 +123  NB: Colour code counters if children use place value counters. 	400 + 30 + 5 100 + 20 + 3 500 + 50 + 8 = 558 As children progress, they will move from the expanded to the compacted method. 435 + 123 558 Then introduce decimals (money)
	Column method with regrouping (3 digit numbers)	Concrete 	Pictorial 	100 + 40 + 6 500 + 20 + 7 600 + 70 + 3 = 673 As the children progress, they will move from the expanded to the compacted method. 146 + 527 673

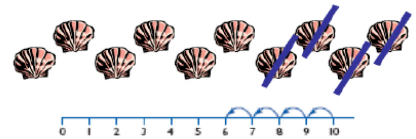
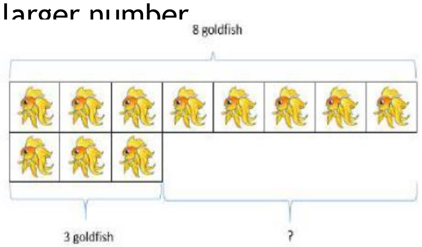
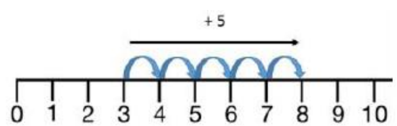
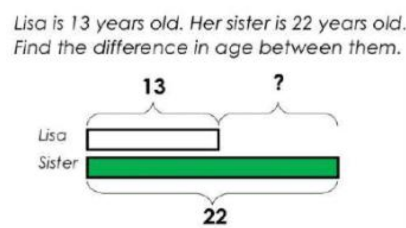
Woodlands Calculation Policy
Addition

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 4	Column method with/without regrouping (4 digit numbers)	Follow Year 3 examples up to 4 digits		
Year 5/6	Column method with regrouping	Consolidate understanding using numbers with more than 4 digits and extend by adding numbers with up to 3 decimals.		

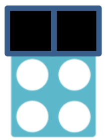
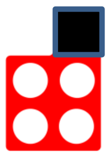
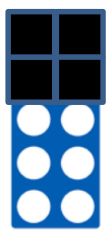
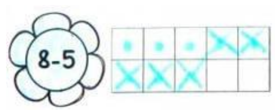
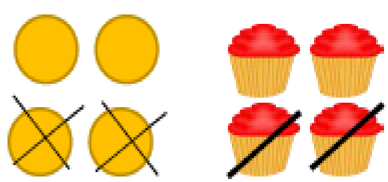
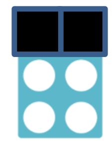
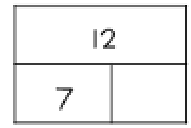
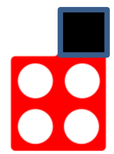
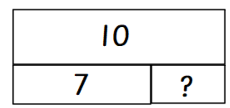
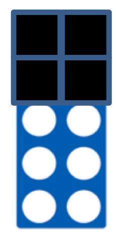
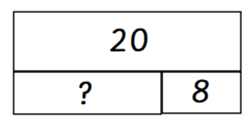
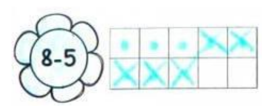
Woodlands Calculation Policy
Subtraction

	Objectives/stages in development	Concrete	Pictorial	Abstract
Reception	Counting back in ones verbally	Begin to count backwards in familiar contexts such as number rhymes or stories 10,9,8,7... Continue the count back in ones from any given number	10,9,8,7...	
	Subtract two single digit numbers Find one less	 (4 – 1 = 3)   	 Three teddies take away two teddies leaves one teddy   	3 - 1 = 2

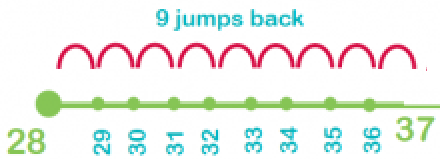
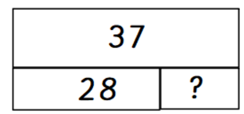
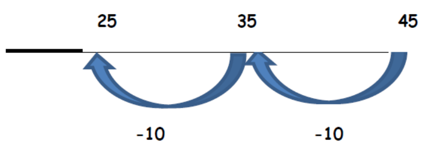
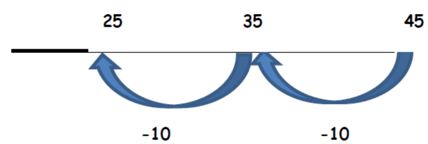
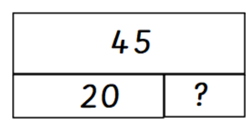
Woodlands Calculation Policy
Subtraction

	Objectives/ stages in developme nt	Concrete	Pictorial	Abstract
Year 1	Counting Back (numbers up to 20)	 13 – 4 = 9	  	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. (Apply use of the number line)
	Finding the difference (numbers up to 20)	Children are shown to count on from the smaller number to the larger number.   Placing Numicon tiles on top of one another.	Children are shown to count on from the smaller number to the larger number.   	Hannah has 8 goldfish. Helen has 3 goldfish. Find the difference between the number of goldfish the girls have. (Apply use of the number line)

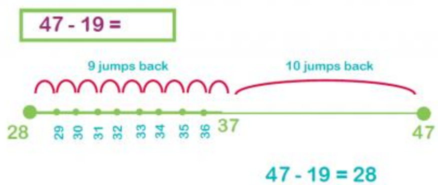
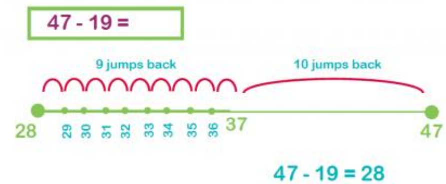
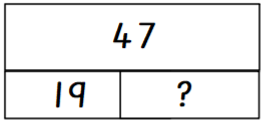
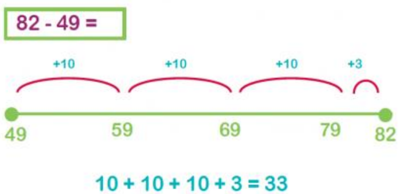
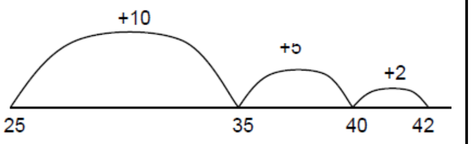
Woodlands Calculation Policy
Subtraction

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 1/2	Taking away one and two digit numbers to10 / 20 **NB:** Including number bonds to 10 / 20.	 6 - 2 = 4  5 - 1 = 4  10 - 4 = 6  	       	5 - 1 = 4 19 - 5 = 14 15 - 11 = 4

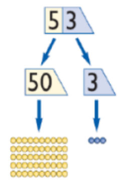
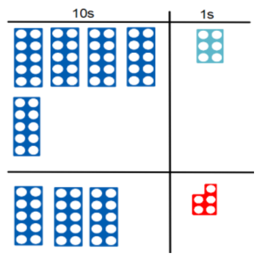
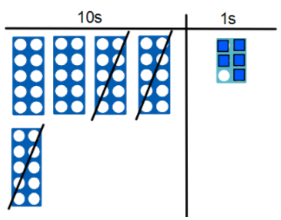
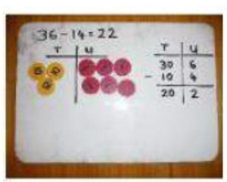
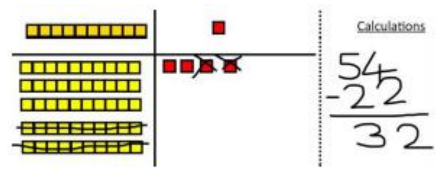
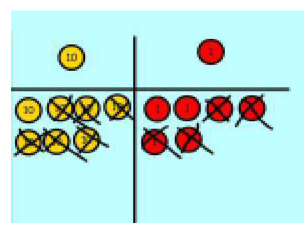
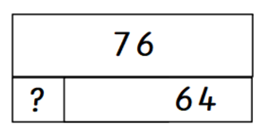
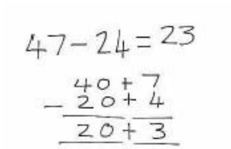
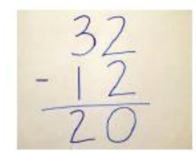
Woodlands Calculation Policy
Subtraction

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 2	Counting back 2 digit number and ones	 37 – 28 = 9	 37 – 28 = 9 	37 – 28 = 9
	Counting back 2 digit number and 10		 	45 - 20

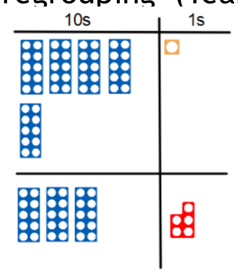
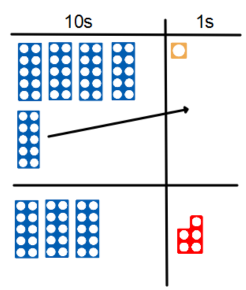
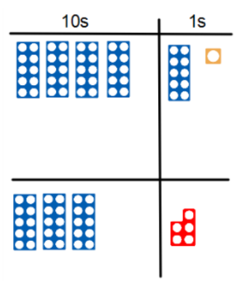
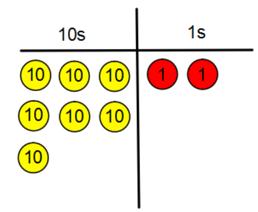
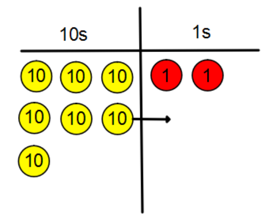
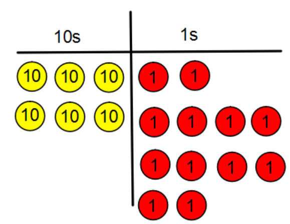
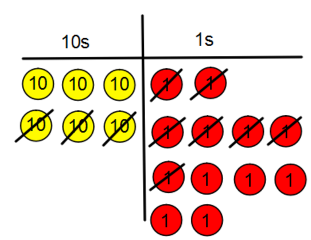
Woodlands Calculation Policy
Subtraction

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 2	Counting back 2 digit number from a 2 digit number		 	47 - 19
	Finding the difference		 10 + 5 + 2 = 17	82 - 49

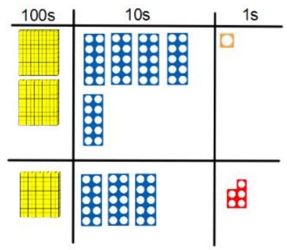
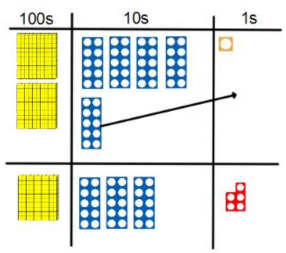
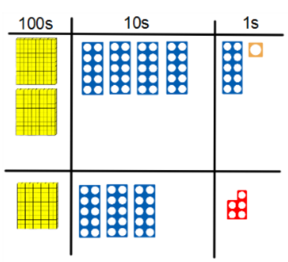
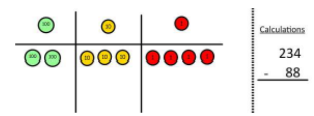
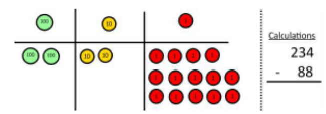
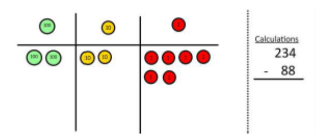
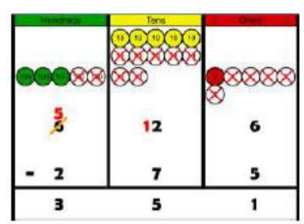
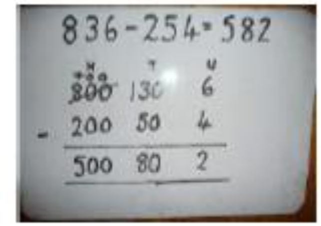
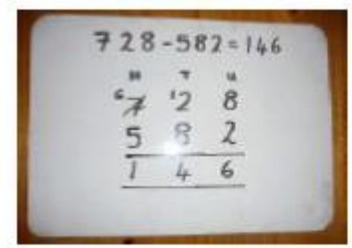
Woodlands Calculation Policy
Subtraction

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 2	Column method without regrouping	 Begin to partition numbers to takeaway.  56 – 35 = 21  	  76 - 64 	 This will lead to a clear written column subtraction. 

Woodlands Calculation Policy
Subtraction

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 2	Column method with regrouping	**<u>NB</u>**: Continue to reinforce column method without regrouping (Year 2)  $\begin{array}{r} 51 \\ -35 \\ \hline \end{array}$  $\begin{array}{r} {}^{4}\cancel{5}\,1 \\ -3\,5 \\ \hline \end{array}$  $\begin{array}{r} {}^{4}\cancel{5}\,{}^{1}1 \\ -3\,5 \\ \hline \end{array}$	   	$\begin{array}{rcr} {}^{30}\cancel{40} & + & {}^{10+}3 \\ -\ 20 & + & 7 \\ \hline 10 & + & 6 \end{array}$

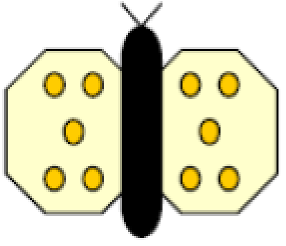
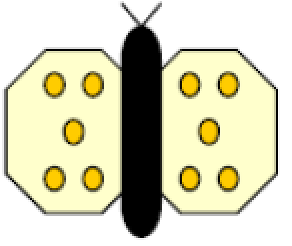
Woodlands Calculation Policy
Subtraction

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 3	Colum Method with Regrouping	**NB**: Continue to reinforce column method without regrouping (Year 2 and 3).  $\begin{array}{r} 251 \\ -\ 135 \\ \hline \end{array}$  $\begin{array}{r} 2\overset{4}{\cancel{5}}1 \\ -\ 135 \\ \hline \end{array}$  $\begin{array}{r} 2\overset{4}{\cancel{5}}{}^{1}1 \\ -\ 135 \\ \hline \end{array}$	Make the larger number with the place value counters  Start with the ones, can I take away 8 from 4 easily? I need to exchange 1 of my tens for 10 ones.  Now I can subtract my ones.  **NB:** Above method can be used as a concrete method  	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals. $\begin{array}{r} 2\,\overset{5}{\cancel{6}}\,\overset{12}{\cancel{3}}\,.\,\overset{1}{0} \\ -\ \ 2\,6\,.\,5 \\ \hline 2\,3\,6\,.\,5 \end{array}$

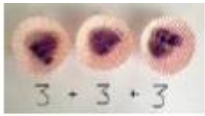
Woodlands Calculation Policy
Subtraction

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 4	Methods should be reinforced in later year groups using the points made to the right.	• use this method for larger numbers (to at least 4 digits) • use this method to subtract numbers with up to 2 decimal places. • solve subtraction problems involving measures and money.		
Year 5 Year 6		• subtract decimals (including those which do not have the same amount of decimals digits). • solve subtraction problems involving measures and money. • use as an inverse operation to check addition calculations.		

Woodlands Calculation Policy
Multiplication

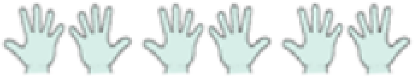
	Objectives/stages in development	Concrete	Pictorial	Abstract
Reception	Know doubles of numbers			$5 + 5 = 10$

Woodlands Calculation Policy
Multiplication

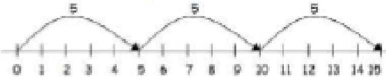
<table>
<tr><th></th><th>Objectives/stages in development</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Year 1/2</td>
<td>Repeated addition</td>
<td>




Use different objects to add equal groups.




</td>
<td>
There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?

2 + 2 + 2 = 6


5 + 5 + 5 = 15


<table>
<tr><td colspan="3">12</td></tr>
<tr><td>4</td><td>4</td><td>4</td></tr>
</table>
</td>
<td>
Write addition sentences to describe objects and pictures.

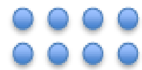
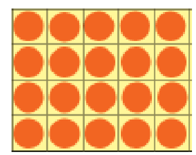
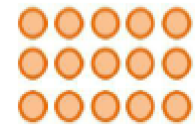
2 + 2 + 2 = 6

2 + 2 + 2 + 2 + 2 = 10
2 × 5 = 10
2 multiplied by 5
5 pairs
5 hops of 2

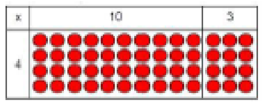
5 + 5 + 5 + 5 + 5 + 5 = 30
5 × 6 = 30
5 multiplied by 6
6 groups of 5
6 hops of 5

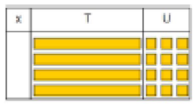
10p + 10p + 10p + 10p + 10p = 50p
10p × 5 = 50p
5 hops of 10
</td>
</tr>
</table>

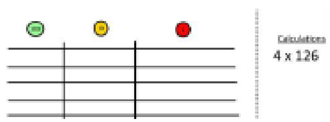
Woodlands Calculation Policy
Multiplication

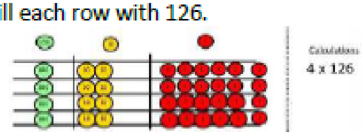
	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 1 / 2	Arrays – showing commutative multiplication	Create arrays using counters/cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.  4 × 2 = 8 2 × 4 = 8  2 × 4 = 8 4 × 2 = 8 Link arrays to area of rectangles. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Woodlands Calculation Policy
Multiplication

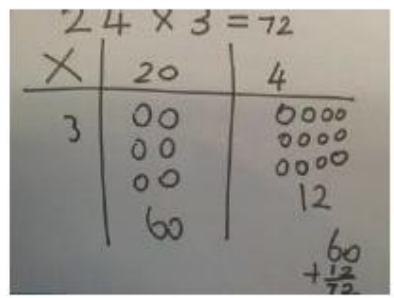
<table>
<tr><th></th><th>Objectives/stages in development</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Year 3/4</td>
<td>Grid Method</td>
<td>
Show the link with arrays to first introduce the grid method.


4 rows of 10
4 rows of 3
Move on to using Base 10 to move towards a more compact method.


4 rows of 13
Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.


Fill each row with 126.


Add up each column, starting with the ones making any exchanges needed.

4 × 126 = 504
</td>
<td>
Children can represent the work they have done with place value counters in a way that they understand.
They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.


<table>
<tr><td colspan="3">72</td></tr>
<tr><td>24</td><td>24</td><td>24</td></tr>
</table>
</td>
<td>
Start with multiplying by one digit numbers and showing the clear addition alongside the grid.
<table>
<tr><td>x</td><td>30</td><td>5</td></tr>
<tr><td>7</td><td>210</td><td>35</td></tr>
</table>
210 + 35 = 245
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.
<table>
<tr><td></td><td>10</td><td>8</td></tr>
<tr><td>10</td><td>100</td><td>80</td></tr>
<tr><td>3</td><td>30</td><td>24</td></tr>
</table>
<table>
<tr><td>X</td><td>1000</td><td>300</td><td>40</td><td>2</td></tr>
<tr><td>10</td><td>10000</td><td>3000</td><td>400</td><td>20</td></tr>
<tr><td>8</td><td>8000</td><td>2400</td><td>320</td><td>16</td></tr>
</table>
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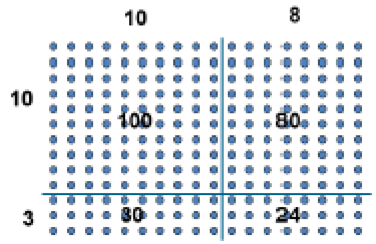
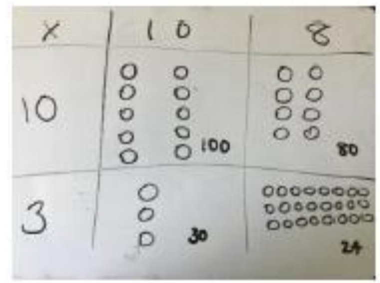
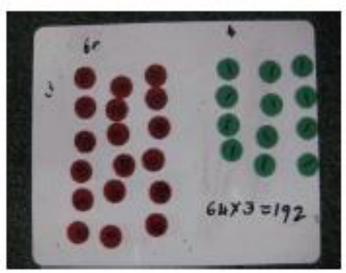
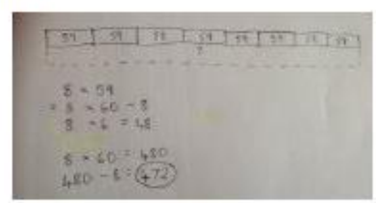
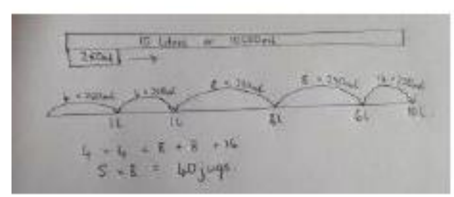
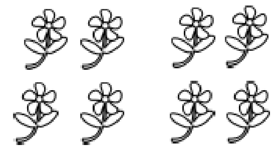
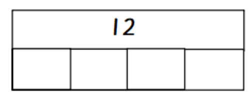
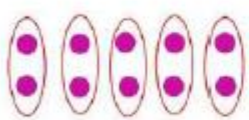
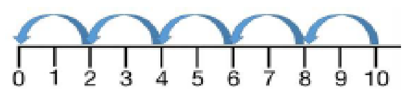
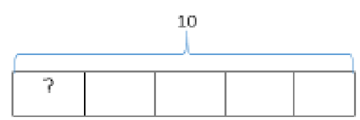
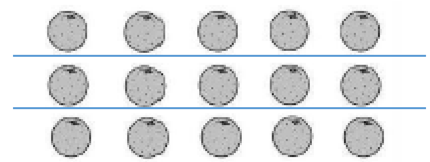
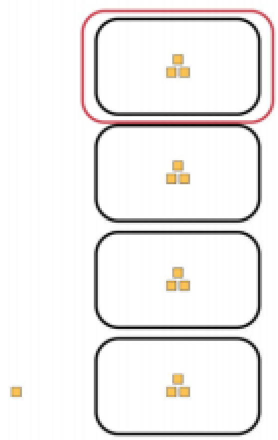
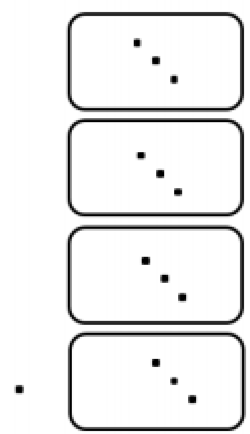
Woodlands Calculation Policy
Multiplication

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 3/4	Expanded Method (2 or 3 digit number by 1 digit number)	Show the link with arrays to first introduce the expanded method. 		Start with long multiplication, reminding the children about lining up their numbers clearly in columns. 18 x 13 24 (3 x 8) 30 (3 x 10)) 80 (10 x 8) 100 (10 x 10) 234
Year 5/6	Expanded Method (4 digit number by 1 or 2 digit number)	Children can continue to be supported by place value counters at the stage of multiplication.  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer. 7 4 x 6 3 1 2 2 1 0 2 4 0 + 4 2 0 0 4 6 6 2 This moves to the more compact method. 1 3 4 2 x 1 8 1 3 4 2 0 1 0 7 3 6 2 4 1 5 6

Woodlands Calculation Policy
Division

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 1/2	Sharing	I have 8 cubes, can you share them equally between two people?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4  	Share 8 buns between two people. 8 ÷ 2 = 4 
	Grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  10 ÷ 5 = ? 5 x ? = 10	10 ÷ 5 = 2 Divide 10 into 5 groups. How many are in each group?

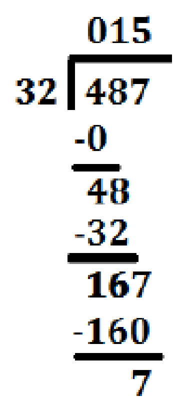
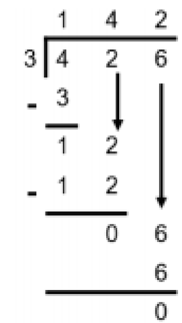
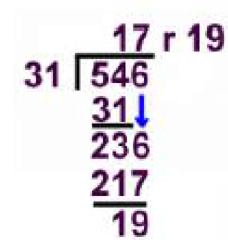
Woodlands Calculation Policy
Division

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 3/4	Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	 Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 5 x 3 = 15 3 x 5 = 15 15 ÷ 5 = 3 15 ÷ 3 = 5
	division			3 4 │ 13 - 12 1 13 ÷ 4 = 3 r 1

Woodlands Calculation Policy
Division

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 5/6	Division with remainders			$\begin{array}{r\|l} & 1\ \ 3 \\ \hline 4 & 5\ {}^{1}2 \end{array}$ 52 ÷ 4 = 13 $\begin{array}{r\|l} & 1\ \ 4\ \ 2 \\ \hline 3 & 4\ {}^{1}2\ \ 6 \end{array}$ 426 ÷ 3 = 142
	Short division with remainders			Move onto divisions with a remainder. Once children understand remainders, begin to express as a fraction or decimal according to the context. $\begin{array}{r\|l} & 8\ \ 6\ \ r\ 2 \\ \hline 5 & 4\ \ 3\ \ {}^{3}2 \end{array}$ $\begin{array}{r\|l} & 1\ \ 8\ \ 6\ \ 1/5 \\ \hline 5 & 9\ {}^{4}3\ {}^{3}1 \end{array}$ $\begin{array}{r\|l} & 1\ \ 4\ .\ 6 \\ \hline 3\ 5 & 5\ \ 1\ {}^{16}1\ .\ {}^{21}0 \end{array}$

Woodlands Calculation Policy
Division

	Objectives/stages in development	Concrete	Pictorial	Abstract
Year 6	Long division			Children will use long division to divide numbers with up to 4 digits by 2 digit numbers.   426 ÷ 3 = 142 

Language that children should become familiar with:

Addition: add addition Plus And count on more sum total altogether increase regrouping

Subtraction: subtract take away minus count back less fewer difference between regrouping, exchange (not borrow)

Multiplication: lots of, groups of, times multiply multiplication multiple product once, twice, three times array, row, column double repeated addition

Division: lots of groups of share group halve half divide division divided by remainder factor quotient divisible